DIFFERENTIAL EQUATIONS

EXERCISE 1.6

Problems solved by;

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Solution Ex 2.6

Q16 Here. $P = (1+3x^{-1})$.

$\Rightarrow P = 1 + 3/x \;\Rightarrow h = \int P\,dx = x + 3\ln x.$

& $e^{h} = e^{x+3\ln x} = e^{x}.e^{\ln x^{3}} = x^{3}e^{x}.$

& $e^{-h} = e^{-(x+3\ln x)} = e^{-x}.e^{\ln x^{-3}} = x^{-3}e^{-x}.$

$r = x+2.$

$\therefore\; y = x^{-3}e^{-x}\left[\int x^{3}e^{x}(x+2)\,dx + c\right].$

$\Rightarrow y = x^{-3}e^{-x}\left[\int (x^{4}e^{x} + 2x^{3}e^{x})\,dx + c\right].$

$\Rightarrow y = x^{-3}e^{-x}\left[\int A\,dx + \int B\,dx + c\right]$ —— (1)

Now $\int A\,dx = \int x^{4}e^{x}dx.$

$\Rightarrow x^{4}e^{x} - \int e^{x}(4x^{3})dx = x^{4}e^{x} - \left(4e^{x}x^{3} - 4\int e^{x}(3x^{2})dx\right)$

$= x^{4}e^{x} - 4e^{x}x^{3} + 12\int e^{x}x^{2}dx.$

$= x^{4}e^{x} - 4e^{x}x^{3} + 12\left(e^{x}x^{2} - \int e^{x}2x\,dx\right)$

$= x^{4}e^{x} - 4e^{x}x^{3} + 12\left(e^{x}x^{2} - 2e^{x}x + 2\int e^{x}dx\right).$

$= x^{4}e^{x} - 4e^{x}x^{3} + 12(e^{x}x^{2} - 2xe^{x} + 2e^{x}).$

$= x^{4}e^{x} - 4e^{x}x^{3} + 12x^{2}e^{x} - 24xe^{x} + 24e^{x}.$

$= e^{x}(x^{4} - 4x^{3} + 12x^{2} - 24x + 24).$

And For $\int B\,dx$. we use.

d/dx		$\int$
$2x^{3}$	$+$	e^{x}
$6x^{2}$	$-$	e^{x}
$12x$	$+$	e^{x}
12	$-$	e^{x}
0		e^{x}

$\int B\,dx = \int 2x^{3}e^{x}dx.$

$\Rightarrow \int B\,dx = e^{x}(2x^{3} - 6x^{2} + 12x - 12)$

putting in (1)

$\Rightarrow y = x^{-3}e^{-x}\left[e^{x}(x^{4} - 4x^{3} + 12x^{2} - 24x + 24) + e^{x}(2x^{3} - 6x^{2} + 12x - 12) + c\right]$

$\Rightarrow y = x^{-3}\left[x^{4} - 4x^{3} + 12x^{2} - 24x + 24 + 2x^{3} - 6x^{2} + 12x - 12 + ce^{-x}\right].$

$\Rightarrow y = x - 4 + 12x^{-1} - 24x^{-2} + 24x^{-3} + 2 - 6x^{-1} + 12x^{-2} - 12x^{-3} + \ldots$

$y(1) = e - 1$

$\therefore e - 1 = 1 - 2 + 6 - 12 + 12 + ce^{-1}$

$\Rightarrow e = 6 + c/e$

$\Rightarrow e = \frac{6e + c}{e} \Rightarrow c = e^2 - 6e.$

putting in ②

$\Rightarrow y = x - 2 + 6x^{-1} - 12x^{-2} + 12x^{-3} + (e^2 - 6e)x^{-3}e^{x}.$

Check

$y' = 1 - 6x^{-2} + 24x^{-3} - 36x^{-4} - 3(e^2 - 6e)x^{-4}e^{x} + (e^2 - 6e)x^{-3}e^{x}.$

Question is

$y' + (1 + 3x^{-1})y = x + 2$

L.H.S

$= 1 - 6x^{-2} + 24x^{-3} - (e^2 - 6e)x^{-3}e^{x} - 36x^{-4} - 3(e^2 - 6e)x^{-4}e^{x}$

$+ (1 + 3x^{-1})(x - 2 + 6x^{-1} - 12x^{-2} + 12x^{-3} + (e^2 - 6e)x^{-3}e^{x})$

$= 1 - \cancel{6x^{-2}} + \cancel{24x^{-3}} - \cancel{(e^2 - 6e)x^{-3}e^{x}} - \cancel{36x^{-4}} - \cancel{3(e^2 - 6e)x^{-4}e^{x}}$

$+ x - 2 + \cancel{6x^{-1}} - \cancel{12x^{-2}} + \cancel{12x^{-3}} + \cancel{(e^2 - 6e)x^{-3}e^{x}} + 3 - \cancel{6x^{-1}}$

$+ \cancel{18x^{-2}} - \cancel{36x^{-3}} + \cancel{36x^{-4}} + \cancel{3(e^2 - 6e)x^{-4}e^{x}}$

$= 2 + x = R.H.S$

$\therefore \boxed{L.H.S = R.H.S}$

$y' = 1 + y^2, \; (y(0) = 0).$

$\Rightarrow \frac{dy}{dx} = 1 + y^2 \Rightarrow \frac{1}{1+y^2}\frac{dy}{dx} = 1.$

$\Rightarrow \int \frac{1}{1+y^2}\,dy = \int dx.$

$\Rightarrow \tan^{-1} y = x + c$

$\Rightarrow y = \tan(x + c)$ — ① Now $(y(0) = 0$ ...

$\Rightarrow y' - y\tan x = 0.$

Q18 here $P = -\tan x$, hence $h = \int P\,dx = \int -\tan x\,dx.$

$\Rightarrow h = \ln\cos x.$

So $e^{h} = \cos x$ and $e^{-h} = \cos^{-1}x = \sec x.$

$\therefore y = \sec x\left[\int \cos x\,(0)\,dx + c\right].$

$\Rightarrow y = c\sec x$ —(1) Given that $y(\pi) = 2.$

$\Rightarrow (2) = c\sec\pi \Rightarrow c = -2.$ putting in (1).

$\Rightarrow y = -2\sec x.$

<u>Check</u> $y' = -2\sec x\tan x.$

& $y' - y\tan x = -2\sec x\tan x + 2\sec x\tan x = 0$

(hence correct)

(19) $y' + 3y = \sin x$, $y(\pi/2) = 0.3.$

here $P = 3$, hence $h = \int 3\,dx = 3x.$

So $e^{h} = e^{3x}$ & $e^{-h} = e^{-3x}.$

$\therefore y = e^{-3x}\left[\int e^{3x}\sin x\,dx + c\right]$ —(1).

Now let $\int e^{3x}\sin x\,dx = I$

$\Rightarrow I = \frac{e^{3x}}{3}\sin x - \int \frac{e^{3x}}{3}\cos x\,dx.$

$\Rightarrow I = \frac{e^{3x}}{3}\sin x - \frac{1}{3}\left(\frac{e^{3x}}{3}\cos x - \int \frac{e^{3x}}{3}(-\sin x)\,dx\right).$

$\Rightarrow I = \frac{e^{3x}}{3}\sin x - \frac{1}{9}e^{3x}\cos x - \frac{1}{9}\int e^{3x}\sin x\,dx.$

$\Rightarrow \left(1 + \frac{1}{9}\right)I = \frac{e^{3x}}{3}\sin x - \frac{1}{9}e^{3x}\cos x \quad \because \int e^{3x}\sin x\,dx = I$

$\Rightarrow \frac{10}{9}I = \frac{e^{3x}}{3}\sin x - \frac{1}{9}e^{3x}\cos x.$

$\Rightarrow I = \frac{3}{10}e^{3x}\sin x - \frac{1}{10}e^{3x}\cos x$, putting in (1)

$\Rightarrow y = e^{-3x}\left[0.3e^{3x}\sin x - 0.1e^{3x}\cos x + c\right].$

$\Rightarrow y = 0.3\sin x - 0.1\cos x + c$ —(2)

$\Rightarrow c = 0$. putting in ②.

$\therefore y = 0.3\sin x - 0.1\cos x$ Ans

(20) $y' + 6x^2 y = e^{-2x^3}/x^2$, $y(1) = 0$.

Here $P = 6x^2 \Rightarrow h = \int P\,dx = 2x^3$.

so $e^h = e^{2x^3}$ & $e^{-h} = e^{-2x^3}$.

$\therefore y = e^{-2x^3}\left[\int e^{2x^3}\frac{e^{-2x^3}}{x^2}dx + c\right]$

$\Rightarrow y = e^{-2x^3}\left[\int x^{-2}dx + c\right]$

$\Rightarrow y = e^{-2x^3}\left[-x^{-1} + c\right]$ — ① now Given that $y(1)=0$

so ① $\Rightarrow 0 = e^{-2}(-1+c)$.

$\Rightarrow c = 1$ putting in ①

$\Rightarrow y = e^{-2x^3}(x^{-1} + 1)$. Ans

(22) $xy' + 4y = 8x^4$, $y(1) = 2$.

$\Rightarrow y' + \frac{4y}{x} = 8x^3$.

Here $P = 4/x \Rightarrow h = \int P\,dx = 4\ln x$. & so

$e^h = x^4$ and $e^{-h} = x^{-4}$. , $r = 8x^3$.

$\therefore y = x^{-4}\left[\int x^4 \cdot 8x^3\,dx + c\right]$.

$\Rightarrow y = x^{-4}\left[\int 8x^7\,dx + c\right] \Rightarrow y = x^{-4}(x^8 + c)$

$\Rightarrow y = x^4 + cx^{-4}$ — ① now Given that $y(1) = 2$.

$\Rightarrow 2 = 1 + c$.

$\Rightarrow c = 1$ putting in ①

$\Rightarrow y = x^4 + x^{-4}$. Ans

(21) $y' = 1 + y^2$, $y(0) = 0$.

$\frac{dy}{dx} = 1 + y^2 \Rightarrow \frac{1}{1+y^2}\frac{dy}{dx} = 1$

$\Rightarrow \int \frac{1}{1+y^2}dy = \int dx \Rightarrow \tan^{-1}y = x + c$ — ①

$\Rightarrow y = \tan x$.

$u = y^{1-a}$

REDUCTION OF NON LINEAR EQUATIONS TO LINEAR FORM

Reduce to linear form & solve the following equations (Some are the Bernoulli's Equation, others become linear if you take x as the unknown function and y as the independent variable)

(31) $y' + 2y = y^2$.

let $u = y^{-a+1}$ here $a = 2$.

$\therefore u_{(x)} = y^{-1} \Rightarrow u' = -y^{-2} \cdot \frac{dy}{dx} \Rightarrow u' = -y^{-2} y'$.

$\Rightarrow u' = -y^{-2}(y^2 - 2y) = -1 + 2y^{-1}$

$\Rightarrow u' = 2/y - 1 \Rightarrow u' - 2y^{-1} = -1$

$\Rightarrow u' - 2u = -1 \quad (\because y^{-1} = u)$.

using the eq(4) from Book

here $p = -2 \Rightarrow h = -2x$; $e^{h} = e^{-2x}$ & $e^{-h} = e^{2x}$.

$\Rightarrow u = e^{2x}\left[\int e^{-2x}(-1)\,dx + c\right]$.

$\Rightarrow u = e^{2x}\left[\frac{e^{-2x}}{2} + c\right]$. $\Rightarrow u = \frac{1}{2} + ce^{2x}$.

putting $u = y^{-1}$.

$\frac{1}{2} + ce^{2x}$ Ans

$\Rightarrow y = \frac{2}{1+2ce^{2x}}$

(32) $y' + y = -x/y \Rightarrow y' + y = -xy^{-1}$

Let $u = y^{1-a}$ here $a = -1$.

$\therefore u = y^{1+1} \Rightarrow u = y^2 \Rightarrow u' = 2yy'$.

$\Rightarrow u' = 2y(-x/y - y) = -2x - 2y^2$.

$\Rightarrow u' = -2x - 2y^2$.

$\Rightarrow \boxed{u' + 2u = -2x}$ This is the Reduced linear equation $\because u = y^2$.

Here $p = 2$.

$\Rightarrow h = \int p\,dx = 2x$ so $e^h = e^{2x}$ & $e^{-h} = e^{-2x}$.

$r = -2x$

$\therefore u = e^{-2x}\left[\int e^{2x}(-2x)\,dx + c\right]$.

$\Rightarrow u = e^{-2x}[I + c]$ —— (1)

Now

$I = -2\int e^{2x} x\,dx \Rightarrow I = -2\left[\frac{e^{2x}}{2}x - \int \frac{e^{2x}}{2}\,dx\right]$.

$\Rightarrow I = \frac{-xe^{2x}}{1} + \frac{e^{2x}}{2}$. putting in (1)

$\Rightarrow u = e^{-2x}\left[\left(-xe^{2x} + \frac{e^{2x}}{2}\right) + c\right]$.

$\Rightarrow u = -x + \frac{1}{2} + ce^{-2x}$.

Now $u = y^2$.

$\therefore y^2 = -x + \frac{1}{2} + ce^{-2x}$

$\Rightarrow y = \sqrt{-x + \frac{1}{2} + ce^{-2x}}$ Ans

(33) $y' + \frac{1}{3}y = \frac{1}{3}(1-2x)y^4$.

Let $u = y^{1-a}$ here $a = 4$.

$\therefore u = y^{-3} \Rightarrow u' = -3y^{-4}y'$.

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$\Rightarrow \{u' - u = -(1-2x)\}$ $\because u = y^{-3}$.
Reduced linear form.

Here $p = -1$ hence $h = -x$, $e^{h} = e^{-x}$ & $e^{-h} = e^{x}$.
$r = -(1-2x)$

$\therefore u = e^{x}\left[\int e^{-x}(-1+2x)\,dx + c\right]$.

$\Rightarrow u = e^{x}\left[-\int e^{-x}\,dx + \int e^{-x}(2x)\,dx + c\right]$.

$\Rightarrow u = e^{x}\left[e^{-x} + \frac{2e^{-x}}{-1}(x) - \int \frac{e^{-x}}{-1}(2)\,dx + c\right]$.

$\Rightarrow u = e^{x}\left[e^{-x} - 2xe^{-x} + 2\frac{e^{-x}}{-1} + c\right]$.

$\Rightarrow u = e^{x}\left[e^{-x} - 2xe^{-x} - 2e^{-x} + c\right]$.

$\Rightarrow u = e^{x}\left[-2xe^{-x} - e^{-x} + c\right]$.

$\Rightarrow u = -2x - 1 + ce^{x}$.
putting $u = 1/y^3$

$\Rightarrow y^3 = \frac{-1}{(2x + 1 - ce^{x})}$ Ans

(34) $y' = \frac{\tan y}{(x-1)}$

$\Rightarrow \frac{dy}{dx} = \frac{\tan y}{x-1} \Rightarrow \frac{1}{\tan y}\cdot\frac{dy}{dx} = \frac{1}{x-1}$

Integrating b/s with respect to x.

$\Rightarrow \int \frac{1}{\tan y}\,dy = \int \frac{1}{(x-1)}\,dx \Rightarrow \int \cot y\,dy = \int \frac{1}{x-1}\,dx$.

$\Rightarrow \ln \sin y = \ln(x-1) + \ln c \Rightarrow \sin y = c(x-1)$

$\Rightarrow y = \sin^{-1}(c(x-1))$.

Check $y' = \frac{c}{\sqrt{1 - c^2(x-1)^2}}$. put $c = \sin y/_{x-1}$

$\Rightarrow y' = \frac{\sin y/(x-1)}{\quad} = \frac{\sin y/(x-1)}{\quad} = \sin y \cdot \frac{1}{(x-1)\ldots} = \tan y$

Q# $\frac{dx}{dy} = 6e^{y} - 2x \Rightarrow x' + 2x = 6e^{y}$ —(1)

Eq-(1) is linear in x. so.

here $P = 2$, $\Rightarrow h = 2y$, $e^{h} = e^{2y}$ & $e^{-h} = e^{-2y}$

$r = 6e^{y}$

$\therefore x = e^{-2y}\left[\int (e^{2y} 6e^{y})\, dy + c\right]$.

$\Rightarrow x = e^{-2y}\left[\int 6e^{3y} dy + c\right]$

$\Rightarrow x = e^{-2y}\left[2e^{3y} + c\right] = 2e^{y} + ce^{-2y}$.

(Ans)

Q# 36 $y'(\sinh 3y - 2xy) = y^2$.

$\Rightarrow \frac{\sinh 3y - 2xy}{y^2} = \frac{1}{y'}$.

$\Rightarrow \frac{\sinh 3y}{y^2} - \frac{2x}{y} = \frac{dx}{dy}$. $\Rightarrow x' + \frac{2}{y}(x) = \frac{\sinh 3y}{y^2}$ —(1)

Note that the Eq (1) is linear in x.

here $P = 2/y \Rightarrow h = \int P dy = \int 2/y\, dy = 2\ln y$.

$e^{h} = e^{2\ln y} = y^{2}$ & $e^{-h} = e^{-2\ln y} = y^{-2}$.

& $r = \frac{\sinh 3y}{y^2}$.

$\therefore x = y^{-2}\left[\int y^{2} \frac{\sinh 3y}{y^{2}}\, dy + c\right]$.

$\Rightarrow x = y^{-2}\left[\int \sinh 3y\, dy + c\right]$.

$\Rightarrow x = y^{-2}\left[\frac{\cosh 3y}{3} + c\right]$.

$\Rightarrow x = \frac{\cosh 3y + 3c}{\quad}$

Q37 $\therefore u = y^2 \Rightarrow u' = 2yy' \Rightarrow u' = 2y(xy' - xy)$.

$\Rightarrow u' = 2x - 2xy^2$.

$\Rightarrow u' = 2x - 2xu \Rightarrow u' + 2xu = 2x$ —— (1)

The equation (1) is linear in u

where $P = 2x \Rightarrow h = x^2$ and $e^h = e^{x^2}$ & $e^{-h} = e^{-x^2}$.

$r = 2x$.

$\therefore u = e^{-x^2}\left[\int e^{x^2}(2x)\,dx + c\right]$.

$\Rightarrow u = e^{-x^2}[I + c]$. —— (2)

now $I = \int e^{x^2} 2x\,dx$. we can calculate this by

put $x^2 = t \Rightarrow dt = 2x\,dx$.

$\therefore I = \int e^t\,dt = e^t = e^{x^2}$ putting in (2)

$\Rightarrow u = e^{-x^2}[e^{x^2} + c]$.

$\Rightarrow u = 1 + ce^{-x^2}$.

put $u = y^2$.

$\therefore y^2 = 1 + ce^{-x^2}$. (Ans)

Q39 What will happen in example 2 in the text if we replace cost by $e^{-0.1t}$ cost?

Total salt content in the inflow $= 50(1 + e^{-0.1t}\cos t)$

Total salt content in the outflow $= 50\, y/1000 = 0.05y$.

Let $y(t)$ is the salt content in the tank.

The rate of change is therefore:

$y' = dy/dt$ = Inflow − Outflow

$\Rightarrow y' = 50(1 + e^{-0.1t}\cos t) - 0.05y$.

or $y' + 0.05y = 50(1 + e^{-0.1t}\cos t)$.

$$y = e^{-0.05t}\left[\int e^{0.05t}\,50(1+\cos t\, e^{-0.1t})\,dt + c\right].$$

$$\text{so } y = e^{-0.05t}\left[\int (50e^{0.05t} + 50e^{-0.05t}\cos t)\,dt + c\right]. \quad \text{--- (a)}$$

Let $I = \int 50\, e^{-0.05t} \cos t\, dt.$

$$= 50\frac{e^{-0.05t}}{-0.05}\cos t - 50\int \frac{e^{-0.05t}}{-0.05}(-\sin t)\,dt.$$

$$= -1000\, e^{-0.05t}\cos t - 1000\int e^{-0.05t}\sin t\, dt$$

$$= -1000\, e^{-0.05t}\cos t - 1000\left[\frac{e^{-0.05t}}{-0.05}\sin t - \int \frac{e^{-0.05t}}{-0.05}\cos t\, dt\right]$$

$$\Rightarrow I = -1000\, e^{-0.05t}\cos t + 20{,}000\, e^{-0.05t}\sin t - 400 I$$

$$\Rightarrow 401 I = -1000\, e^{-0.05t}\cos t + 20000\, e^{-0.05t}\sin t.$$

$$\Rightarrow I = -2.493\, e^{-0.05t}\cos t + 49.88\, e^{-0.05t}\sin t.$$

Also let $H = \int 50 e^{0.05t}\,dt.$

$$\Rightarrow H = 50\frac{e^{0.05t}}{(0.05)} = 1000\, e^{0.05t}$$ putting both I and H in Eq — (a)

$$\Rightarrow y = e^{-0.05t}\left[1000\, e^{0.05t} - 2.49 e^{(-0.05t+0.05t)}\cos t + 49.88\, e^{(-0.05t+0.05t)}\sin t + c\right]$$

$$\Rightarrow y = 1000 + (2.49\cos t + 49.88\sin t)\, e^{-0.1t} + c e^{-0.05t} \quad \text{— (b)}$$

Now from initial condition i-e
$y(0) = 200$ putting in (b)

$$\Rightarrow 200 = 1000 - (2.49\cos 0 + 49.88\sin 0)\, e^{-0.1t} + c e^{0}$$

$\Rightarrow$ ~~[illegible]~~ $= -802.5$

$\Rightarrow C = -797.506.$

putting in (b).

$\Rightarrow y = 1000 - 2.49 \cos t\, e^{-0.1t} + 49.88 \sin t\, e^{-0.1t} - 797.5\, e^{-0.05t}$

Ans

Q#40 $a = b = k = 1$ and $y(0) = 2$.

$$y' = a - b\cos\left(\frac{2\pi t}{24}\right) - ky.$$

$$\Rightarrow y' + ky = a - b\cos\left(\frac{\pi t}{12}\right).$$

here $p = k$ so $h = kt$, $e^{h} = e^{kt}$ & $e^{-h} = e^{-kt}$

$$\therefore y = e^{-kt}\left[\int e^{kt}\left(a - b\cos\left(\frac{\pi t}{12}\right)\right) dt + c\right]$$

$$\Rightarrow y = e^{-kt}\left[\frac{a e^{kt}}{k} - b\int e^{kt}\cos\left(\frac{\pi t}{12}\right) dt + c\right] \text{—— (1)}$$

Let $I = \int e^{kt}\cos\left(\frac{\pi t}{12}\right) dt.$

$$\Rightarrow I = \frac{e^{kt}}{k}\cos\left(\frac{\pi t}{12}\right) - \int \frac{e^{kt}}{k}\left(-\sin\left(\frac{\pi t}{12}\right)\cdot\frac{\pi}{12}\right) dt$$

$$\Rightarrow I = \frac{e^{kt}}{k}\cos\left(\frac{\pi t}{12}\right) + \frac{\pi}{12k}\int e^{kt}\sin\left(\frac{\pi t}{12}\right) dt.$$

$$\Rightarrow I = \frac{e^{kt}}{k}\cos\left(\frac{\pi t}{12}\right) + \frac{\pi}{12k}\left[\frac{e^{kt}}{k}\sin\left(\frac{\pi t}{12}\right) - \int \frac{e^{kt}}{k}\left(\cos\frac{\pi t}{12}\right)\frac{\pi}{12}\, dt\right]$$

$$\Rightarrow I = \frac{e^{kt}}{k}\cos\left(\frac{\pi t}{12}\right) + \frac{\pi}{12k^2} e^{kt}\sin\left(\frac{\pi t}{12}\right) - \frac{\pi^2}{(12k)^2} I.$$

$$\Rightarrow \left(1 + \frac{\pi^2}{(12k)^2}\right) I = \frac{e^{kt}}{k}\cos\left(\frac{\pi t}{12}\right) + \frac{\pi}{12k^2} e^{kt}\sin\left(\frac{\pi t}{12}\right)$$

$$\Rightarrow \left(\frac{(12k)^2 + \pi^2}{(12k)^2}\right) I = \frac{e^{kt}}{k}\cos\left(\pi t/12\right) + \pi/12k^2\, e^{kt}\sin\left(\pi t/12\right)$$

$$\Rightarrow I = \frac{(12)^2 k}{\ldots} e^{kt}\cos\left(\pi t/12\right) + \frac{12\pi}{(12k)^2 + \pi^2} e^{kt}\sin\left(\pi t/12\right)$$

$\frac{22}{84} = \frac{11}{42}$

putting in ①

$$\Rightarrow y = e^{-kt}\left[\frac{ae^{kt}}{k} - b\left(\frac{144k}{(12k)^2+\pi^2}\cos\left(\frac{\pi t}{12}\right) + \frac{12\pi}{(12k)^2+\pi^2}\sin\left(\frac{\pi t}{12}\right)\right) + c\right]$$

$$\Rightarrow y = e^{-kt}\left[\frac{ae^{kt}}{k} - \frac{144bke^{kt}}{(12k)^2+\pi^2}\cos\left(\frac{\pi t}{12}\right) + \frac{12\pi b e^{kt}}{(12k)^2+\pi^2}\sin\left(\frac{\pi t}{12}\right) + c\right] \quad \text{—②}$$

putting $a = b = k = 1$ and $y(0) = 2$

$$\Rightarrow 2 = e^{0}\left[1 - \frac{144}{144+\pi^2} + c\right].$$

$$\Rightarrow c = 1 + \frac{144}{144+\pi^2} = \frac{144+\pi^2+144}{144+\pi^2}$$

putting in ②

$$\Rightarrow y = e^{-t}\left[\frac{e^t}{1} - \frac{144e^t}{144+\pi^2}\cos\left(\frac{\pi t}{12}\right) + \frac{12\pi e^t}{144+\pi^2}\sin\left(\frac{\pi t}{12}\right) + \frac{288+\pi^2}{144+\pi^2}\right].$$

Ans