

DIFFERENTIAL EQUATIONS

EXERCISE 2.5

Problems solved by;

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general solution.

Find the general solution using factorization.

$$\textcircled{5} (D^2 - D - 2)y = 0$$

$$\Rightarrow (D^2 - 2D + D - 2)y = 0 \Rightarrow [D(D-2) + 1(D-2)]y = 0$$

$$\Rightarrow (D-2)(D+1)y = 0$$

$$\Rightarrow (D-2)(Dy + y) = 0 \Rightarrow (D-2)(y' + y) = 0$$

$$\Rightarrow Dy' + Dy - 2y' - 2y = 0$$

$$\Rightarrow y'' + y' - 2y = 0$$

hence our factorization is permissible.

The solution of

$$(D-2)y = 0, \quad (D+1)y = 0$$

$$\Rightarrow Dy - 2y = 0, \quad Dy + y = 0$$

$$\Rightarrow y' - 2y = 0, \quad y' + y = 0$$

$$\Rightarrow \ln y = -2x + \ln c, \quad \ln y = -x + \ln c$$

$$\Rightarrow y_1 = c_1 e^{-2x}, \quad y_2 = c_2 e^{-x}$$

hence

$$y = y_1 + y_2 = (c_1 e^{-2x} + c_2 e^{-x})$$

Ans

$$\textcircled{6} (9D^2 + 6D + 1)y = 0$$

$$\Rightarrow (9D^2 + 3D + 3D + 1)y = 0 \Rightarrow (3D(3D+1) + 1(3D+1))y = 0$$

$$\Rightarrow ((3D+1)(3D+1))y = 0, \text{ we get two equal factors.}$$

so the solution is

$$(3D+1)y = 0 \Rightarrow 3y' + y = 0 \Rightarrow \ln y = -\frac{x}{3} + \ln c$$

$$\Rightarrow y_1 = c_1 e^{-x/3}$$

hence the general solution is

$$y = (c_1 + c_2 x) e^{-x/3}$$

Ans

$$\textcircled{7} (D^2 - 4D)y = 0$$

$$\Rightarrow (D(D-4))y = 0$$

$$\Rightarrow Dy = 0, \quad (D-4)y = 0$$

$$\Rightarrow y_1 = c_1, \quad \ln y_2 = 4x + \ln c_2 \Rightarrow y_2 = c_2 e^{4x}$$

hence the general sol is

$$y = y_1 + y_2 = c_1 + c_2 e^{4x}$$

Ans

$$\Rightarrow (5D - 1)y = 0 \quad \Rightarrow \frac{dy}{dx} = \frac{1}{5}y$$

$$(8) \Rightarrow ((5D+1)(5D-1))y = 0$$

$$\Rightarrow (5D+1)y = 0, \quad (5D-1)y = 0$$

$$\Rightarrow 5Dy + y = 0, \quad 5Dy - y = 0$$

$$\Rightarrow \ln y_1 = -\frac{x}{5} + \ln c_1, \quad \ln y_2 = \frac{x}{5} + \ln c_2$$

$$\Rightarrow y_1 = c_1 e^{-x/5}, \quad y_2 = c_2 e^{x/5}$$

hence the general solution is

$$y = y_1 + y_2 = c_1 e^{-x/5} + c_2 e^{x/5}$$

$$(9) (D^2 + 2kD + k^2)y = 0$$

$$\Rightarrow (D^2 + kD + kD + k^2)y = 0$$

$$\Rightarrow (D(D+k) + k(D+k))y = 0$$

$$\Rightarrow (D+k)(D+k)y = 0$$

we get two equal factors.

$$\text{so } (D+k)y = 0 \Rightarrow y' + ky = 0$$

$$\Rightarrow \ln y_1 = -kx + \ln c_1$$

$$\Rightarrow y_1 = c_1 e^{-kx}$$

$$\text{and thus } y_2 = u y_1 = \int \frac{1}{y_1} dx = c_2 x e^{-kx}$$

hence the general solution is

$$y = (c_1 + c_2 x) e^{-kx} \quad \underline{\text{Ans}}$$

$$(10) (D^2 + \lambda(\lambda-1)D - \lambda^3)y = 0$$

$$\Rightarrow (D^2 + \lambda^2 D - \lambda D - \lambda^3)y = 0$$

$$\Rightarrow (D(D + \lambda^2) - \lambda(D + \lambda^2))y = 0$$

$$\Rightarrow (D - \lambda)y = 0, \quad (D + \lambda^2)y = 0$$

$$\Rightarrow Dy_1 - \lambda y_1 = 0, \quad Dy_2 + \lambda^2 y_2 = 0$$

$$\Rightarrow y_1' = \lambda y_1, \quad y_2' = -\lambda^2 y_2$$

$$\Rightarrow \ln y_1 = \lambda x + \ln c_1, \quad \ln y_2 = -\lambda^2 x + \ln c_2$$

$$\Rightarrow y_1 = c_1 e^{\lambda x} \quad \& \quad y_2 = c_2 e^{-\lambda^2 x}$$

$$y = y_1 + y_2 = c_1 e^{1x} + c_2 e^{-1x}$$

Ans

(11) $(64D^2 + 16D + 1)y = 0$
 $\Rightarrow (64D^2 + 8D + 8D + 1)y = 0$
 $\Rightarrow (8D(8D + 1) + 1(8D + 1))y = 0$
 $\Rightarrow ((8D + 1)(8D + 1))y = 0$

equal factors and thus equal roots

So $(8D + 1)y = 0$
 $\Rightarrow 8Dy + y = 0 \Rightarrow 8y' + y = 0$
 $\Rightarrow y' = -y/8 \Rightarrow \ln y = -\frac{1}{8}x + \ln c_1$
 $\Rightarrow y = y_1 = c_1 e^{-x/8}$

Hence the general solution is

$$y = (c_1 + c_2 x) e^{-x/8}$$

Ans

(12) $(2D^2 + D)y = 0$

$\Rightarrow (D(2D + 1))y = 0$
 $\Rightarrow Dy = 0, (2D + 1)y = 0$
 $\Rightarrow y_1 = c_1, 2y' + y_2 = 0$
 $\Rightarrow \ln y_2 = -\frac{1}{2}x + c_2$
 $\Rightarrow y_2 = c_2 e^{-x/2}$

Hence the general solution is

$$y = c_1 + c_2 e^{-x/2}$$

Ans

(13) $(10D^2 + 12D + 3.6)y = 0$

$\Rightarrow (10D^2 + 6D + 6D + 3.6)y = 0$
 $\Rightarrow ((2D(5D + 3) + 1.2(5D + 3))y = 0$
 $\Rightarrow ((5D + 3)(2D + 1.2))y = 0$

$\Rightarrow (5D + 3)y = 0$

We have $D_1 = -3/5$ and $D_2 = -1.2/2 = -0.6$

$$y = c_1 e^{-3/5x} + c_2 x e^{-1.2/2x}$$

$$y = (c_1 + c_2 x) e^{-0.6x}$$

Ans